

Exchangeability and Data Analysis

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SUMMARY

The term ‘exchangeability’ was introduced by de Finetti in the context of personal probability to describe a particular sense in which quantities treated as random variables in a probability specification are thought to be similar. We believe, however, that judgments of similarity are more primitive than those of probability and are at the heart of all statistical activities, including those for which probability specifications are absent or contrived. In this paper, we give a definition of exchangeability in a descriptive context, which extends de Finetti’s concept to a wider domain. Our objective is to analyse the logic of judgments of exchangeability (or similarity, or homogeneity), to clarify the roles of context and data analysis in these judgments. We give several examples to illustrate the nature of these judgments in description, inference and prediction. We use this discussion to clarify the extent to which judgments of similarity in inference and prediction can be based on data, and the extent to which they must rely on pure faith. Our discussion is a contribution to the emerging theory of data analysis, the as yet largely atheoretical and informal process that precedes and supports formal statistical activities.

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1. INTRODUCTION

Statistical methods are concerned with combining information from different observational units, and with making inferences from the resulting summaries to prospective measurements on the same or other units. These operations will be useful only when the units to be combined are judged to be *similar* (comparable or homogeneous). This paper focuses on the *logic* of judgments of similarity, particularly the interacting roles of contextual information and data manipulation or display. To clarify their roles, we propose a modest formalism that identifies the elements of a judgment of similarity. Although this formalism may remind readers of significance tests, it is distinct, and we use it to analyse the logic of similarity judgments, not to construct procedures.

The term ‘exchangeability’ was introduced by de Finetti (1930, 1974), who defined it in the context of personalistic probability specifications. He used it to describe a sense in which random quantities in such a specification are judged to be similar. We believe, however, that judgments of similarity involve concepts more primitive than probability, and that these judgments are central to preliminary activities that all statisticians must perform, even though probability specifications are absent or contrived at such a preliminary stage. We propose to extend the use of the term

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